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REPORT NO. 58

QUASI-SQUARE HOLE WITH OPTIMUM SHAPE IN
AN INFINITE PLATE SUBJECTED TO IN-PLANE LOADING

BY A. J. DURELLI AND K. RAJAIAH

REPORT NO. 59

OPTIMIZATION OF INNER AND OUTER BOUNDARIES
OF BEAMS AND PLATES WITH HOLES

BY A. J. DURELLI AND K. RAJAIAH

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UNIVERSITY OF MARYLAND
COLLEGE PARK, MD. 20742

TWO OPTIMIZATION PROBLEMS

REPORTS 58 AND 59

In reports 55, 56 and 57 applications of the newly developed optimization technique have been presented. They dealt with circular holes in square plates and in cylindrical shells.

The two reports presented here deal also with optimization of the shape of structural elements. In the first one (report 58) the optimum geometry is given for a "quasi-square" hole in an infinite plate. Here again stress concentrations have been eliminated and the geometry is relatively easy to obtain practically, the loading applied to the plate can be uniaxial or of biaxiality 1:-1. In the second report (number 59) both inner and outer boundaries of some particular beams and plates are optimized. In both reports the information is presented in form to be used directly by designers. In all the cases studied, savings of weight and increase in strength are large.

The abstracts of the previously published reports, prepared with O.N.R. support, are presented only once, before the test of the two reports. The O.N.R. reports distribution list is also presented only once, at the end. A Report Documentation Page follows each individual report.

Previous Technical Reports to the Office of Naval Research

1. A. J. Durelli, "Development of Experimental Stress Analysis Methods to Determine Stresses and Strains in Solid Propellant Grains"--June 1962. Developments in the manufacturing of grain-propellant models are reported. Two methods are given: a) cementing routed layers and b) casting.

2. A. J. Durelli and V. J. Parks, "New Method to Determine Restrained Shrinkage Stresses in Propellant Grain Models"--October 1962. The birefringence exhibited in the curing process of a partially restrained polyurethane rubber is used to determine the stress associated with restrained shrinkage in models of solid propellant grains partially bonded to the case.

3. A. J. Durelli, "Recent Advances in the Application of Photoelasticity in the Missile Industry"--October 1962. Two- and three-dimensional photoelastic analysis of grains loaded by pressure and by temperature are presented. Some applications to the optimization of fillet contours and to the redesign of case joints are also included.

4. A. J. Durelli and V. J. Parks, "Experimental Solution of Some Mixed Boundary Value Problems"--April 1964. Means of applying known displacements and known stresses to the boundaries of models used in experimental stress analysis are given. The application of some of these methods to the analysis of stresses in the field of solid propellant grains is illustrated. The presence of the "pinching effect" is discussed.

5. A. J. Durelli, "Brief Review of the State of the Art and Expected Advance in Experimental Stress and Strain Analysis of Solid Propellant Grains"--April 1964. A brief review is made of the state of the experimental stress and strain analysis of solid propellant grains. A discussion of the prospects for the next fifteen years is added.

6. A. J. Durelli, "Experimental Strain and Stress Analysis of Solid Propellant Rocket Motors"--March 1965. A review is made of the experimental methods used to strain-analyze solid propellant rocket motor shells and grains when subjected to different loading conditions. Methods directed at the determination of strains in actual rockets are included.

7. L. Ferrer, V. J. Parks and A. J. Durelli, "An Experimental Method to Analyze Gravitational Stresses in Two-Dimensional Problems"--October 1965. Photoelasticity and moiré methods are used to solve two-dimensional problems in which gravity-stresses are present.

8. A. J. Durelli, V. J. Parks and C. J. del Rio, "Stresses in a Square Slab Bonded on One Face to a Rigid Plate and Shrunk"--November 1965.
A square epoxy slab was bonded to a rigid plate on one of its faces in the process of curing. In the same process the photoelastic effects associated with a state of restrained shrinkage were "frozen-in." Three-dimensional photoelasticity was used in the analysis.

9. A. J. Durelli, V. J. Parks and C. J. del Rio, "Experimental Determination of Stresses and Displacements in Thick-Wall Cylinders of Complicated Shape"--April 1966.
Photoelasticity and moiré are used to analyze a three-dimensional rocket shape with a star shaped core subjected to internal pressure.

10. V. J. Parks, A. J. Durelli and L. Ferrer, "Gravitational Stresses Determined Using Immersion Techniques"--July 1966.
The methods presented in Technical Report No. 7 above are extended to three-dimensions. Immersion is used to increase response.

11. A. J. Durelli and V. J. Parks. "Experimental Stress Analysis of Loaded Boundaries in Two-Dimensional Second Boundary Value Problems"--February 1967.
The pinching effect that occurs in two-dimensional bonding problems, noted in Reports 2 and 4 above, is analyzed in some detail.

12. A. J. Durelli, V. J. Parks, H. C. Feng and F. Chiang, "Strains and Stresses in Matrices with Inserts,"-- May 1967.
Stresses and strains along the interfaces, and near the fiber ends, for different fiber end configurations, are studied in detail.

13. A. J. Durelli, V. J. Parks and S. Uribe, "Optimization of a Slot End Configuration in a Finite Plate Subjected to Uniformly Distributed Load,"--June 1967.
Two-dimensional photoelasticity was used to study various elliptical ends to a slot, and determine which would give the lowest stress concentration for a load normal to the slot length.

14. A. J. Durelli, V. J. Parks and Han-Chow Lee, "Stresses in a Split Cylinder Bonded to a Case and Subjected to Restrained Shrinkage,"--January 1968.
A three-dimensional photoelastic study that describes a method and shows results for the stresses on the free boundaries and at the bonded interface of a solid propellant rocket.

15. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--August 1968.
This report has been written following a trip conducted by the author through several European countries. A list is given of many of the laboratories doing important experimental stress analysis work and of the people interested in this kind of work. An attempt has been made to abstract the main characteristics of the methods used in some of the countries visited.

16. V. J. Parks, A. J. Durelli and L. Ferrer, "Constant Acceleration Stresses in a Composite Body"--October 1968.
Use of the immersion analogy to determine gravitational stresses in two-dimensional bodies made of materials with different properties.

17. A. J. Durelli, J. A. Clark and A. Kochev, "Experimental Analysis of High Frequency Stress Waves in a Ring"--October 1968.
A method for the complete experimental determination of dynamic stress distributions in a ring is demonstrated. Photoelastic data is supplemented by measurements with a capacitance gage used as a dynamic lateral extensometer.

18. J. A. Clark and A. J. Durelli, "A Modified Method of Holographic Interferometry for Static and Dynamic Photoelasticity"--April 1968.
A simplified absolute retardation approach to photoelastic analysis is described. Dynamic isopachics are presented.

19. J. A. Clark and A. J. Durelli, "Photoelastic Analysis of Flexural Waves in a Bar"--May 1969.
A complete direct, full-field optical determination of dynamic stress distribution is illustrated. The method is applied to the study of flexural waves propagating in a urethane rubber bar. Results are compared with approximate theories of flexural waves.

20. J. A. Clark and A. J. Durelli, "Optical Analysis of Vibrations in Continuous Media"--June 1969.
Optical methods of vibration analysis are described which are independent of assumptions associated with theories of wave propagation. Methods are illustrated with studies of transverse waves in prestressed bars, snap loading of bars and motion of a fluid surrounding a vibrating bar.

21. V. J. Parks, A. J. Durelli, K. Chandrashekhara and T. L. Chen, "Stress Distribution Around a Circular Bar, with Flat and Spherical Ends, Embedded in a Matrix in a Triaxial Stress Field"--July 1969.
A Three-dimensional photoelastic method to determine stresses in composite materials is applied to this basic shape. The analyses of models with different loads are combined to obtain stresses for the triaxial cases.

22. A. J. Durelli, V. J. Parks and L. Ferrer, "Stresses in Solid and Hollow Spheres Subjected to Gravity or to Normal Surface Tractions"--October 1969.
The method described in Report No. 10 above is applied to two specific problems. An approach is suggested to extend the solutions to a class of surface traction problems.

23. J. A. Clark and A. J. Durelli, "Separation of Additive and Subtractive Moiré Patterns"--December 1969.
A spatial filtering technique for adding and subtracting images of several gratings is described and employed to determine the whole field of Cartesian shears and rigid rotations.

24. R. J. Sanford and A. J. Durelli, "Interpretation of Fringes in Stress-Holo-Interferometry"--July 1970.
Errors associated with interpreting stress-holo-interferometry patterns as the superposition of isopachics (with half order fringe shifts) and isochromatics are analyzed theoretically and illustrated with computer generated holographic interference patterns.

25. J. A. Clark, A. J. Durelli and P. A. Laura, "On the Effect of Initial Stress on the Propagation of Flexural Waves in Elastic Rectangular Bars"--December 1970.
Experimental analysis of the propagation of flexural waves in prismatic, elastic bars with and without prestressing. The effects of prestressing by axial tension, axial compression and pure bending are illustrated.

26. A. J. Durelli and J. A. Clark, "Experimental Analysis of Stresses in a Buoy-Cable System Using a Birefringent Fluid"--February 1971.
An extension of the method of photoviscous analysis is presented which permits quantitative studies of strains associated with steady state vibrations of immersed structures. The method is applied in an investigation of one form of behavior of buoy-cable systems loaded by the action of surface waves.

27. A. J. Durelli and T. L. Chen, "Displacements and Finite-Strain Fields in a Sphere Subjected to Large Deformations"--February 1972.
Displacements and strains (ranging from 0.001 to 0.50) are determined in a polyurethane sphere subjected to several levels of diametral compression. A 500 lines-per-inch grating was embedded in a meridian plane of the sphere and moiré effect produced with a non-deformed master. The maximum applied vertical displacement reduced the diameter of the sphere by 27 per cent.

28. A. J. Durelli and S. Machida, "Stresses and Strain in a Disk with Variable Modulus of Elasticity"--March 1972
A transparent material with variable modulus of elasticity has been manufactured that exhibits good photoelastic properties and can also be strain analyzed by moiré. The results obtained suggests that the stress distribution in the disk of variable E is practically the same as the stress distribution in the homogeneous disk. It also indicates that the strain fields in both cases are very different, but that it is possible, approximately, to obtain the stress field from the strain field using the value of E at every point, and Hooke's law.

29. A. J. Durelli and J. Buitrago, "State of Stress and Strain in a Rectangular Belt Pulled Over a Cylindrical Pulley"--June 1972.
Two- and three-dimensional photoelasticity as well as electrical strain gages, dial gages and micrometers are used to determine the stress distribution in a belt-pulley system. Contact and tangential stress for various contact angles and friction coefficients are given.

30. T. L. Chen and A. J. Durelli, "Stress Field in a Sphere Subjected to Large Deformations"--June 1972.
Strain fields obtained in a sphere subjected to large diametral compressions from a previous paper were converted into stress fields using two approaches. First, the concept of strain-energy function for an isotropic elastic body was used. Then the stress field was determined with the Hookean type natural stress-natural strain relation. The results so obtained were also compared.

31. A. J. Durelli, V. J. Parks and H. M. Hasseem, "Helices Under Load"--July 1973.
Previous solutions for the case of close coiled helical springs and for helices made of thin bars are extended. The complete solution is presented in graphs for the use of designers. The theoretical development is correlated with experiments.

32. T. L. Chen and A. J. Durelli, "Displacements and Finite Strain Fields in a Hollow Sphere Subjected to Large Elastic Deformations"--September 1973.
The same methods described in No. 27, were applied to a hollow sphere with an inner diameter one half the outer diameter. The hollow sphere was loaded up to a strain of 30 per cent on the meridian plane and a reduction of the diameter by 20 per cent.

33. A. J. Durelli, H. H. Hasseem and V. J. Parks, "New Experimental Method in Three-Dimensional Elastostatics"--December 1973.
A new material is reported which is unique among three-dimensional stress-freezing materials, in that, in its heated (or rubbery) state it has a Poisson's ratio which is appreciably lower than 0.5. For a loaded model, made of this material, the unique property allows the direct determination of stresses from strain measurements taken at interior points in the model.

34. J. Wolak and V. J. Parks, "Evaluation of Large Strains in Industrial Applications"--April 1974.
It was shown that Mohr's circle permits the transformation of strain from one axis of reference to another, irrespective of the magnitude of the strain, and leads to the evaluation of the principal strain components from the measurement of direct strain in three directions.

35. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--April 1975.
Continuation of Report No. 15 after a visit to Belgium, Holland, Germany, France, Turkey, England and Scotland.

36. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Linear and Non-linear Elastic and Plastic Strains in a Plate with a Big Hole Loaded Axially in its Plane"--July 1975.
Strain analysis of the ligament of a plate with a big hole indicates that both geometric and material non-linearity may take place. The strain concentration factor was found to vary from 1 to 2 depending on the level of deformation.

37. A. J. Durelli, V. Pavlin, J. O. Bühler-Vidal and G. Ome, "Elastostatics of a Cubic Box Subjected to Concentrated Loads"--August 1975.
Analysis of experimental strain, stress and deflection of a cubic box subjected to concentrated loads applied at the center of two opposite faces. The ratio between the inside span and the wall thickness was varied between approximately 5 and 121.

38. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Elastostatics of Cubic Boxes Subjected to Pressure"--March 1976.
Experimental analysis of strain, stress and deflections in a cubic box subjected to either internal or external pressure. Inside span-to-wall thickness ratio varied from 5 to 14.

39. Y. Y. Hung, J. D. Hovanesian and A. J. Durelli, "New Optical Method to Determine Vibration-Induced Strains with Variable Sensitivity After Recording"--November 1976.
A steady state vibrating object is illuminated with coherent light and its image slightly misfocused. The resulting specklegram is "time-integrated" as when Fourier filtered gives derivatives of the vibrational amplitude.

40. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Cyclic Stress Studies by Time-Averaged Photoelasticity"--November 1976.
"Time-averaged isochromatics" are formed when the photographic film is exposed for more than one period. Fringes represent amplitudes of the oscillating stress according to the zeroth order Bessel function.

41. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Time-Averaged Shadow Moiré Method for Studying Vibrations"--November 1976.
Time-averaged shadow moiré permits the determination of the amplitude distribution of the deflection of a steady vibrating plate.

42. J. Buitrago and A. J. Durelli, "On the Interpretation of Shadow-Moiré Fringes"--April 1977.
Possible rotations and translations of the grating are considered in a general expression to interpret shadow-moiré fringes and on the sensitivity of the method. Application to an inverted perforated tube.

43. J. der Hovanesian, "18th Polish Solid Mechanics Conference." Published in European Scientific Notes of the Office of Naval Research, in London, England, Dec. 31, 1976.
Comments on the planning and organization of, and scientific content of paper presented at the 18th Polish Solid Mechanics Conference held in Wisla-Jawornik from September 7-14, 1976.

44. A. J. Durelli, "The Difficult Choice,"--May 1977.
The advantages and limitations of methods available for the analyses of displacements, strain, and stresses are considered. Comments are made on several theoretical approaches, in particular approximate methods, and attention is concentrated on experimental methods: photoelasticity, moiré, brittle and photoelastic coatings, gages, grids, holography and speckle to solve two- and three-dimensional problems in elasticity, plasticity, dynamics and anisotropy.

45. C. Y. Liang, Y. Y. Hung, A. J. Durelli and J. D. Hovanesian, "Direct Determination of Flexural Strains in Plates Using Projected Gratings,"--June 1977.
The method requires the rotation of one photograph of the deformed grating over a copy of itself. The moiré produced yields strains by optical double differentiation of deflections. Applied to projected gratings the idea permits the study of plates subjected to much larger deflections than the ones that can be studied with holograms.

46. A. J. Durelli, K. Brown and P. Yee, "Optimization of Geometric Discontinuities in Stress Fields"--March 1978.
The concept of "coefficient of efficiency" is introduced to evaluate the degree of optimization. An ideal design of the inside boundary of a tube subjected to diametral compression is developed which decreases its maximum stress by 25%, at the time it also decreases its weight by 10%. The efficiency coefficient is increased from 0.59 to 0.95. Tests with a brittle material show an increase in strength of 20%. An ideal design of the boundary of the hole in a plate subjected to axial load reduces the maximum stresses by 26% and increases the coefficient of efficiency from 0.54 to 0.90.

47. J. D. Hovanesian, Y. Y. Hung and A. J. Durelli, "New Optical Method to Determine Vibration-Induced Strains With Variable Sensitivity After Recording"--May 1978.
A steady-state vibrating object is illuminated with coherent light and its image is slightly misfocused in the film plane of a camera. The resulting processed film is called a "time-integrated specklegram." When the specklegram is Fourier filtered, it exhibits fringes depicting derivatives of the vibrational amplitude. The direction of the spatial derivative, as well as the fringe sensitivity may be easily and continuously varied during the Fourier filtering process. This new method is also much less demanding than holographic interferometry with respect to vibration isolation, optical set-up time, illuminating source coherence, required film resolution. etc.

48. Y. Y. Hung and A. J. Durelli, "Simultaneous Determination of Three Strain Components in Speckle Interferometry Using a Multiple Image Shearing Camera,"--September 1978
This paper describes a multiple image-shearing camera. Incorporating coherent light illumination, the camera serves as a multiple shearing speckle interferometer which measures the derivatives of surface displacements with respect to three directions simultaneously. The application of the camera to the study of flexural strains in bent plates is shown, and the determination of the complete state of two-dimensional strains is also considered. The multiple image-shearing camera uses an interference phenomena, but is less demanding than holographic interferometry with respect to vibration isolation and the coherence of the light source. It is superior to other speckle techniques in that the obtained fringes are of much better quality.

49. A. J. Durelli and K. Rajaiah, "Quasi-square Hole With Optimum Shape in an Infinite Plate Subjected to In-plane Loading"--January 1979. This paper deals with the optimization of the shape of the corners and sides of a square hole, located in a large plate and subjected to in-plane loads. Appreciable disagreement has been found between the results obtained previously by other investigators. Using an optimization technique, the authors have developed a quasi-square shape which introduces a stress concentration of only 2.54 in a uniaxial field, the comparable value for the circular hole being 3. The efficiency factor of the proposed optimum shape is 0.90. whereas the one of the best shape developed previously was 0.71. The shape also is developed that minimizes the stress concentration in the case of biaxial loading when the ratio of biaxiality is 1:-1.

50. A. J. Durelli and K. Rajaiah, "Optimum Hole Shapes in Finite Plates Under Uniaxial Load,"--February 1979.
This paper presents optimized hole shapes in plates of finite width subjected to uniaxial load for a large range of hole to plate widths (D/W) ratios. The stress concentration factor for the optimized holes decreased by as much as 44% when compared to circular holes. Simultaneously, the area covered by the optimized hole increased by as much as 26% compared to the circular hole. Coefficients of efficiency between 0.91 and 0.96 are achieved. The geometries of the optimized holes for the D/W ratios considered are presented in a form suitable for use by designers. It is also suggested that the developed geometries may be applicable to cases of rectangular holes and to the tip of a crack. This information may be of interest in fracture mechanics.

51. A. J. Durelli and K. Rajaiah, "Determination of Strains in Photoelastic Coatings,"--May 1979
Photoelastic coatings can be cemented directly to actual structural components and tested under field conditions. This important advantage has made them relatively popular in industry. The information obtained, however, may be misinterpreted and lead to serious errors. A correct interpretation requires the separation of the principal strains and so far, this operation has been found very difficult. Following a previous paper by one of the authors, it is proposed to drill small holes in the coating and record the birefringence at points removed from the edge of the holes. The theoretical background of the method is reviewed; the technique necessary to use it is explained and two applications are described. The precision of the method is evaluated and found satisfactory in contradiction to information previously published in the literature.

52. A. J. Durelli and K. Rajaiah, "Optimized Inner Boundary Shapes in Circular Rings Under Diametral Compression,"--June 1979.
Using a method developed by the authors, the configuration of the inside boundary of circular rings, subjected to diametral compression, has been optimized, keeping cleared the space enclosed by the original circular inside boundary. The range of diameters studied was $0.33 \leqslant ID/OD \leqslant 0.7$. In comparison with circular rings of the same ID/OD, the stress concentrations have been reduced by about 30%, the weight has been reduced by about 10% and coefficients of efficiency of about 0.96 have been attained. The maximum values of compressive and tensile stresses on the edge of the hole, are approximately equal, there are practically no gradients of stress along the edge of the hole, and sharp corners exhibit zero stress. The geometries for each ID/OD design are given in detail.

53. A. J. Durelli and K. Rajaiah, "Lighter and Stronger,"--February 1980.
A new method has been developed that permits the direction design of shapes of two-dimensional structures and structural components, loaded in their plane, within specified design constrains and exhibiting optimum distribution of stresses. The method uses photoelasticity and requires a large field diffused light polariscope. Several problems of optimization related to the presence of holes in finite and infinite plates, subjected to uniaxial and biaxial loadings, are solved parametrically.
Some unexpected results have been found: 1) the optimum shape of a large hole in a bar of finite width, subjected to uniaxial load, is "quasi" square, but the transverse boundary has the configuration of a "hat"; 2) for the small hole in the large plate, a "barrel" shape has a lower s.c.f. than the circular hole and appreciably higher coefficient of efficiency; 3) the optimum shape of a tube, subjected to diametral compression, has small "hinges" and is much lighter and stronger than the circular tube. Applications are also shown to the design of dove-tails and slots in turbine blades and rotors, and to the design of star-shaped solid propellant grains for rockets.

54. C. Brémond and A. J. Durelli, "Experimental Analysis of Displacements and Shears at the Surface of Contact Between two Loaded Bodies,"--July 1980.
The displacements which exist at the contact between two loaded bodies depend on the geometry of the surface of contact, the type of the loading and the property of the materials. A method has been developed to determine these displacements experimentally. A grid has been photographically printed on an interior plane of a transparent model of low modulus of elasticity. The displacements were recorded photographically and the analysis was conducted on the photographs of the deformed grids. Shears were determined from the change in angles. The precision of the measurements at the interface is estimated to be plus or minus 0.05mm. Examples of application are given for the cases of loads applied normally and tangentially to a rigid cylindrical punch resting on a semi-infinite solft plate. Important observations can be made on the zones of friction and of slip. The proposed method is three-dimensional and the distributions can be obtained at several interior planes by changing the position of the plane of the grid. The limitations of the method are pointed out. The possibility of using gratings (12 to 40 lpmm) is considered as well as the advantages of using moiré to analyze the displacements.

55. M. Erickson and A. J. Durelli, "Stress Distribution Around a Circular Hole in Square Plates, Loaded Uniformly in the Plane, on two Opposite Sides of the Square"--The complete stress distribution around a circular hole, located in the center of a square plate, has been determined photoelastically for the case of the plate loaded uniformly on two opposite sides. The study was conducted parametrically for a large range of the ratio of the side of the square to the diameter of the hole. The results obtained permit the determination of the stresses for any biaxial condition and verify a previous solution obtained for the case of the pressurized hole. The experimental procedure is briefly described.

56. A. J. Durelli, M. Erickson and K. Rajaiah, " "This paper presents the shapes that will optimize the stress distribution about central holes in square plates subjected to uniform load on two opposite sides of the plate. The study is conducted for a large range of hole to plate widths ratios (D/W). The stress concentration factor for the optimized holes decreased by as much as 21% when compared to the one associated with a circular hole. Simultaneously, the weight of the plate with optimized hole is reduced by as much as 36% as compared to the circular hole. Coefficient of efficiecy of around 0.92 is achieved for all D/W ratios. The geometry of the optimized holes are presented in a form suitable for use by designers.

57. K. Rajaiah and A.J. Durelli, "Optimization of Hole Shapes in Circular Cylindrical Shells under Axial Tension" -- Hole shapes are optimized in circular cylindrical shells subjected to axial load considering only the predominantly large membrane stresses present around the holes. Two-dimensional photoelastic isochromatics obtained with a special-purpose polariscope are utilized for the optimization process. The process leads to a significant decrease in the membrane stress-concentration factor and a modest decrease in weight, thus yielding a considerable increase in strength-to-weight ratio. This paper presents results for certain typical ratios of hole diameter to shell diameter. Previous theoretical and experimental studies for the circular hole have also been verified.

58. A.J. Durelli and K. Rajaiah "Quasi-Square Hole with Optimum Shape in an Infinite Plate Subjected to In-Plane Loading" - This paper deals with the optimization of the shape of the corners and sides of a square hole, located in a large plate and subjected to in-plane loads, with the object of minimizing stress concentrations. Appreciable disagreement has been found between the results obtained previously by other investigators. In this paper new tests have been conducted and discrepancies have been corrected. Using an optimization technique, the authros have developed a quasi square shape which introduces a stress concentration of only 2.54 in a uniaxial field, the comparable value for the circular hole being 3. The efficiency factor of the proposed optimum shape is 0.90 whereas the efficiency factor of the best shape developed previously was 0.71. The shape also is developed that minimizes the stress concentration in the case of biaxial loading when the ratio of biaxiality is 1:-1.

59. A.J. Durelli and K. Rajaiah, "Optimization of Inner and Outer Boundaries of Beams and Plates with Holes" - This paper presents optimized shapes of inner and outer boundaries for three specific problems: A long rectangular plate with a central hole subjected to uniaxial tension, a simply-supported slotted beam subjected to a load uniformly distributed over a small area at the centre, and a square plate with a central hole under uniaxial uniform pressure. The two-dimensional photoelastic method is used for optimization. The results indicate a significant reduction in stress concentration factor or in weight, or in both. The examples presented also include cases where the inner and outer boundary stresses are mutually dependent.

QUASI-SQUARE HOLE WITH OPTIMUM SHAPE IN AN INFINITE PLATE SUBJECTED TO IN-PLANE LOADING

by

A. J. Durelli and K. Rajaiah

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TABLE OF CONTENTS

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Quasi-Square Hole With Optimum Shape in an Infinite Plate Subjected to In-Plane Loading

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This paper deals with the optimization of the shape of the corners and sides of a square hole, located in a large plate and subjected to in-plane loads, with the object of minimizing stress concentrations. Appreciable disagreement has been found between the results obtained previously by other investigators. In this paper new tests have been conducted and discrepancies have been corrected. Using an optimization technique, the authors have developed a quasi square shape which introduces a stress concentration of only 2.54 in a uniaxial field, the comparable value for the circular hole being 3. The efficiency factor of the proposed optimum shape is 0.90 whereas the efficiency factor of the best shape developed previously was 0.71. The shape also is developed that minimizes the stress concentration in the case of biaxial loading when the ratio of biaxiality is 1:-1.

Introduction

The problem of a square hole with rounded corners in an infinite plate subjected to uniaxial loading has attracted the attention of several investigators over the years. Richmond [1] conducted photoelastic tests with three different corner radii and concluded that "a minimum stress concentration factor seems to result for a value of r/D of about 1/6," where r is the corner radius and D the width of hole. He also found that the minimum value of the stress concentration factor (s.c.f.) was less than 3, the value corresponding to the circular hole. Mindlin [2] in the discussion of the paper stated that Richmond's finding was a result of importance. Peterson [3] in his recent monograph on "Stress Concentration Factors" presented the theoretical results obtained by Sobey [4] for rectangular holes with round corners. Comparison of Sobey's values with those of Richmond for square holes shows gross underestimate of the s.c.f. by Richmond (Fig. 1). According to Sobey, the minimum possible stress concentration factor is 2.85 for a corner radius of 0.37D, whereas Richmond reported a minimum of 2.5 for a corner radius of $D/6$. As stated by Peterson [5], Richmond "probably used a small model, and with techniques of that time and the edge effect problem, his results could be considerably in error." The importance of the subject and contradictions among authors made advisable the review of other contributions and conducting new tests. That is the object of this paper. Emphasis will be placed on the optimization of the shape.

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Previous Contributions

Sobey in his review of previous analyses [6-9] that were conducted using the Schwarz-Christoffel transformation of the square with sharp corners as approximation to the square with round corners found that the authors used only two or three terms in the mapping function and "their profile differs considerably in local curvature variation from the ideal profile so that the stress distributions are not very accurate." Sobey used Mushkilishvili's complex variable method [6] but included a large number of terms in the mapping function to get high accuracy for the hole shape and hence for the stress distribution. Isida [10] analyzed the problem of hypotrochoidal hole with four sides which approximates square holes with rounded corners in finite and infinite plates using a perturbation method. His numerical results for a corner radius of 0.125D are lower than Sobey's (Fig. 1).

Savin [11] presented the results obtained by Leknitsky [12] using the conformal mapping technique. For the corner radius 0.2D these results are also lower than those obtained by Sobey (Fig. 1).

Ross [13] reported results of numerous photoelastic experiments on holes and notches in thin plates under uniaxial tension, including those for square holes with rounded corners. His results show a gross overestimate of the s.c.f.'s when compared to Sobey's values (Fig. 1). It would appear that the extrapolation technique used by Ross is not correct (for the circular hole he obtains a s.c.f. of 3.25). Ross also presented results for a barrel-shaped hole proposed earlier by Heywood [14] as an "ideal shape" for holes in inifinite plates under uniaxial tension. The s.c.f. for this case also appears to have been overestimated.

More recently Durelli, Brown and Yee [15] have shown that, similarly to the earlier work for fillets by Durelli et. al., [16-19] hole shapes can also be very effectively optimized by

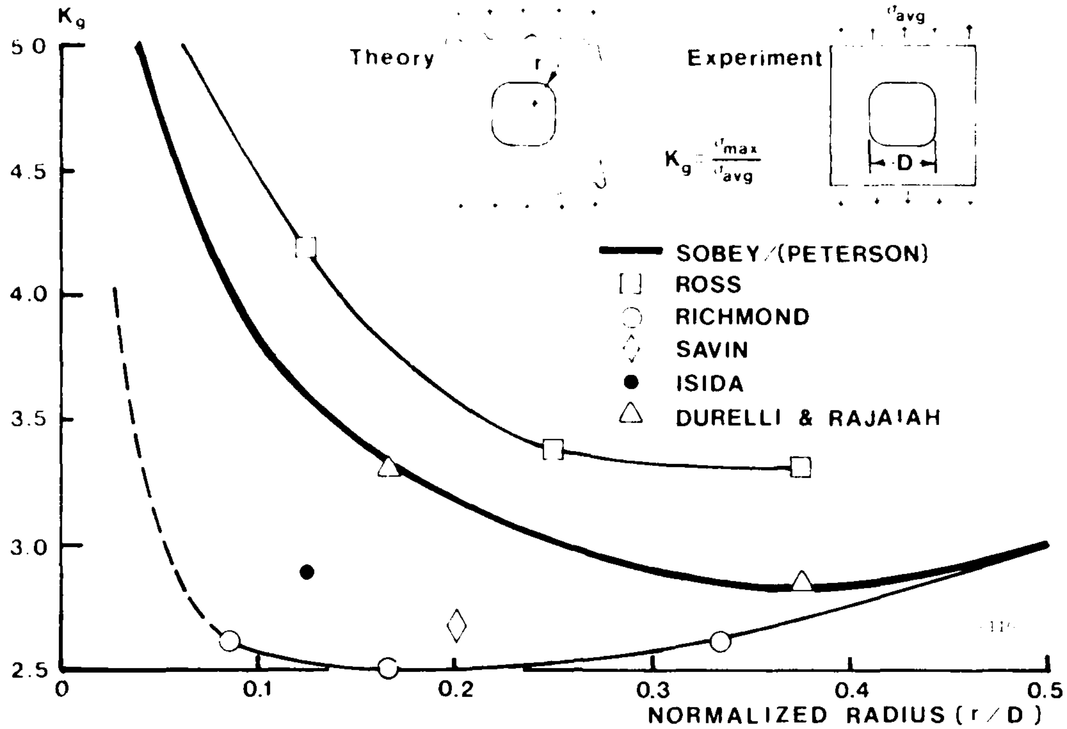

Fig. 1 Stress concentration factors for a square hole with rounded corners in a wide plate subjected to uniaxial loading, obtained by several authors

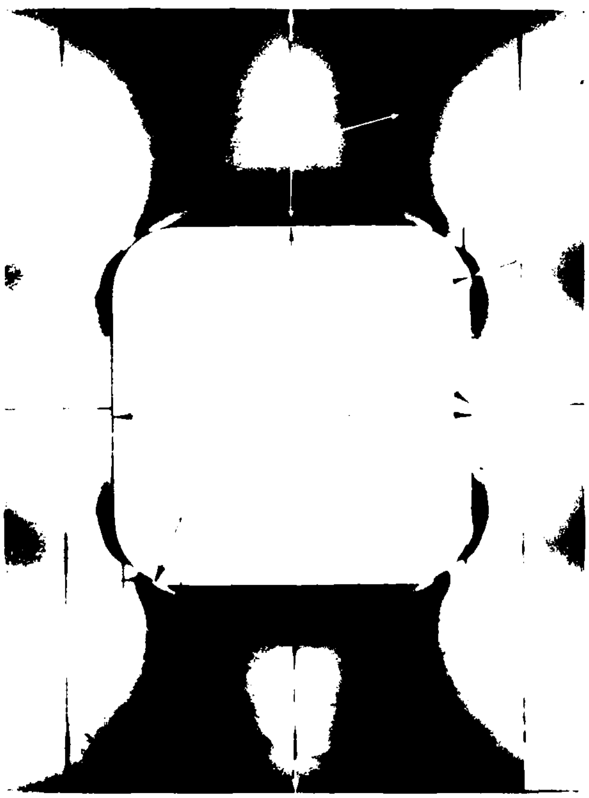

Fig. 2 Stresses at the boundary of a square hole with a rounded corner (r = D/6) in a large plate (D W = 0.136) subjected to uniaxial load

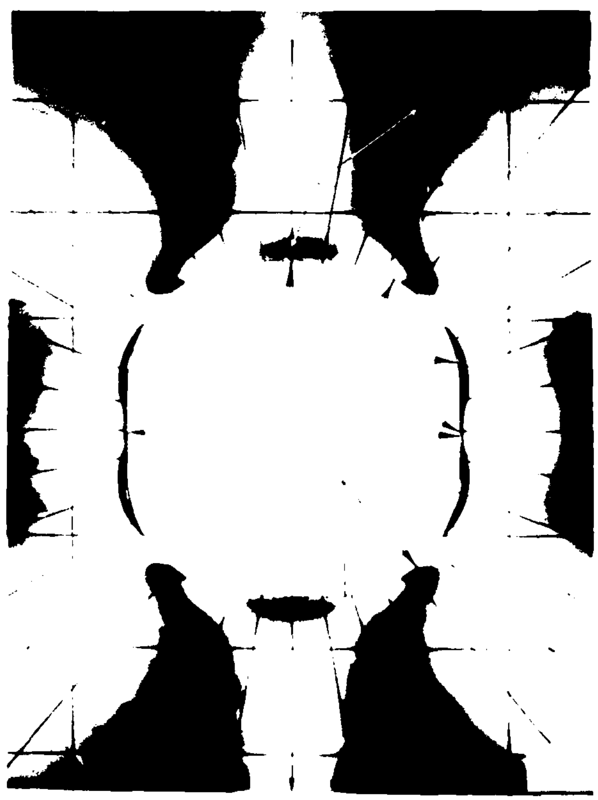

Fig. 3 Stresses at the boundary of a square hole with a rounded corner (r = 0.37D) in a large plate (D W = 0.136) subjected to uniaxial load

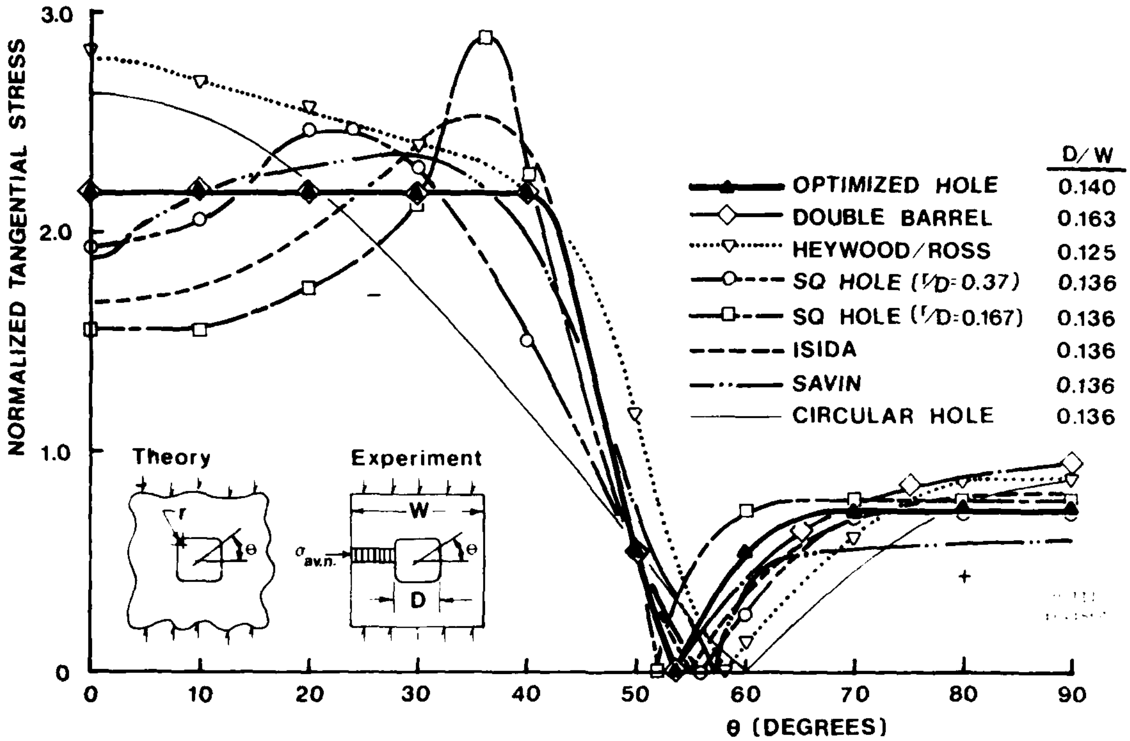

Fig. 4 Distribution of stresses around a quasi-square hole with rounded corners in a large plate subjected to uniaxial loading

using two-dimensional photoelastic techniques. Following that approach, it is shown in the present work that by optimizing the shape of the square hole, a s.c.f. significantly lower than the lowest value given can be achieved. It has also been verified experimentally that the results given by Richmond and Ross are in error

Square Hole with Rounded Corners

Experiments were conducted first on plates with square holes with rounded corners for two different corner radii. Two plates of 11″ × 11″ × 0.272″ (280 × 280 × 2.9mm) with hole size (D) of 1.5″ × 1.5″ (38 × 38mm), one with a corner radius of 0.25″ (6.4mm) equal to (D/6) and the other with 0.555″ (14mm) equal to (0.37D) radius were machined out of Homalite-100. The fringe constant was 151 lb/in-fr (26.2 kN/m-fr). The plates were loaded under uniform compression on two opposite edges. The resulting s.c.f. values are shown in Fig. 1 and the corresponding photoelastic isochromatic patterns are shown in Figs. 2 and 3. The stress distributions around the hole for the two cases, referred to the net area, are included in Fig. 4. It is seen from Fig. 1 that the present results are in close agreement with Sobey's and not so with those of Richmond, Ross, Savin nor Isida. (The s.c.f. given by Peterson and Ross, referred to the gross area, have been referred for comparison purposes to the net area.)

Optimization of the Square Hole

The constraints of the problem are: (a) the inside boundary has to lie inbetween the circle of diameter D and the square of side D; (b) the allowable maximum stress for compression is about three times the allowable maximum stress for tension. To start the optimization process a plate with a hole with a corner radius of 0.37D was selected as this hole exhibits a low s.c.f. Material was removed from the lower stress regions of the boundary, at and near the horizontal axis and at the corner by careful hand filing while the model was under load until an isochromatic fringe coincided with the boundary of the model. The resulting isochromatic pattern is shown in Fig. 5 and the stress distribution around the hole is presented in Fig. 4. For the sake of comparison, Fig. 4 also includes the distributions given by Ross (for Heywood's ideal shape), by Savin for the square hole with 0.2D corner radius, by Isida for the square hole with 0.125D corner radius and also for the circular hole.

The empirically developed geometry has been fitted with a combination of circles of different diameters and common tangents at the points of intersections. The geometry of the optimized shape is shown in Fig. 6.

In an earlier paper [8], it was proposed that the degree of optimizaion be evaluated quantitatively as a coefficient of efficiency, k_{eff}, defined as

$$k_{eff} = \frac{1}{S_2 - S_0} \left\{ \frac{\int_{S_0}^{S_1} \sigma_t^+ \, ds}{\sigma_{all}^+} + \frac{\int_{S_1}^{S_2} \sigma_t^- \, ds}{\sigma_{all}^-} \right\}$$

where σ_{all} represents the maximum allowable stress (the positive and negative superscripts referring to tensile and compressive stresses, respectively), S_0 and S_1 are the limiting points of the segment of the boundary subjected to tensile stresses and S_1 and S_2 are limiting points of the segment of boundary with compressive stresses. The same criterion has been used here too to evaluate the shapes and the results are discussed below.

The Biaxial Case

It has been shown that the optimum shape of a hole in a biaxial field of two loadings of the same sign is an ellipse the eccentricity of which is related to the biaxiality ratio [20]. For

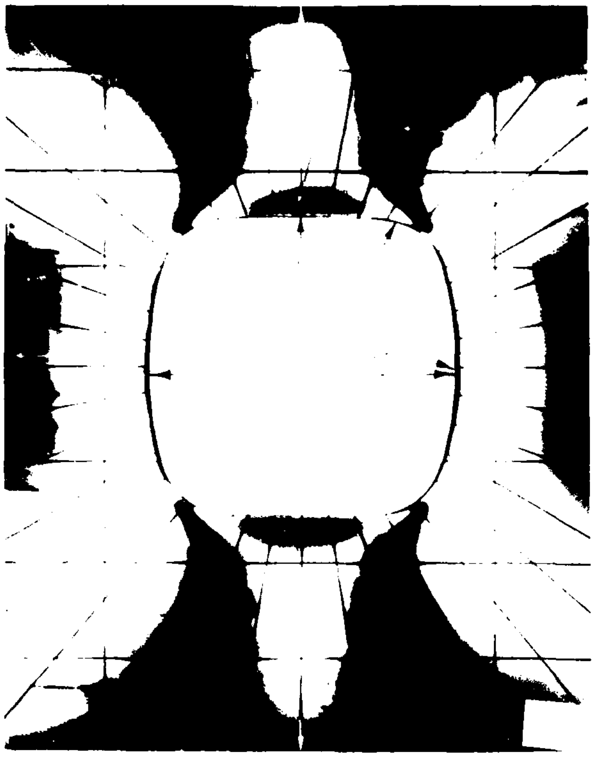

Fig. 5 Stresses at the boundary of an optimized quasi-square hole in large plate (D/W = 0.140) subjected to uniaxial load

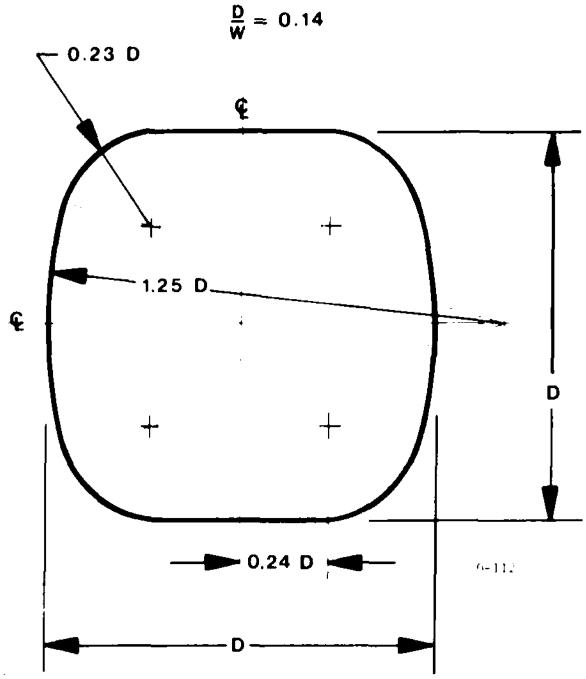

Fig. 6 Optimized geometry of a quasi-square hole associated with the minimun stress concentration factor in a large plate subjected to uniaxial loading

1:1 biaxiality, the shape of the hole is a circle and the s.c.f. is 2. However, for loadings of opposite sign no such simple relation has been found.

The above study suggests that, for a plate under pure shear (1:-1 biaxiality), a doubly symmetric shape with both the longitudinal and the transverse edges of barrel shape (double barrel) would appear to be close to an optimum shape. This case has also been investigated here.

A double barrel shaped hole with the radius of curvature of each edge being 1.25D as in the case of the optimized hole was made in a large Homalite-100 plate and tested. For a uniaxial loading this shape provides a slightly increased stress level on the edge perpendicular to the load while the one on the edge parallel to the load stays the same (Fig. 4 and Fig. 7). However, for the case of shear loading, there is a 10% reduction in s.c.f. (Fig. 8).

The Case of the Notch

It is well-known that the s.c.f. for the case of a semicircular edge notch in a wide plate under uniaxial load is approximately the same as that for a circular hole in a wide plate under uniaxial load [3]. Based on this observation, it is believed that the optimum shape for a notch in a wide plate subjected to uniaxial loading will be approximately the same as the optimum shape developed above for the case of the square hole.

Discussion and Conclusion

By the optimization method followed here, it has been possible to obtain a s.c.f. of 2.54 for the square hole with rounded corners. This value is about 11% lower than the s.c.f. value of 2.85 as given by Peterson following Sobey. The efficiency factor for the optimum shape is 0.90 whereas it is 0.71 for the shape given by Peterson as corresponding to the

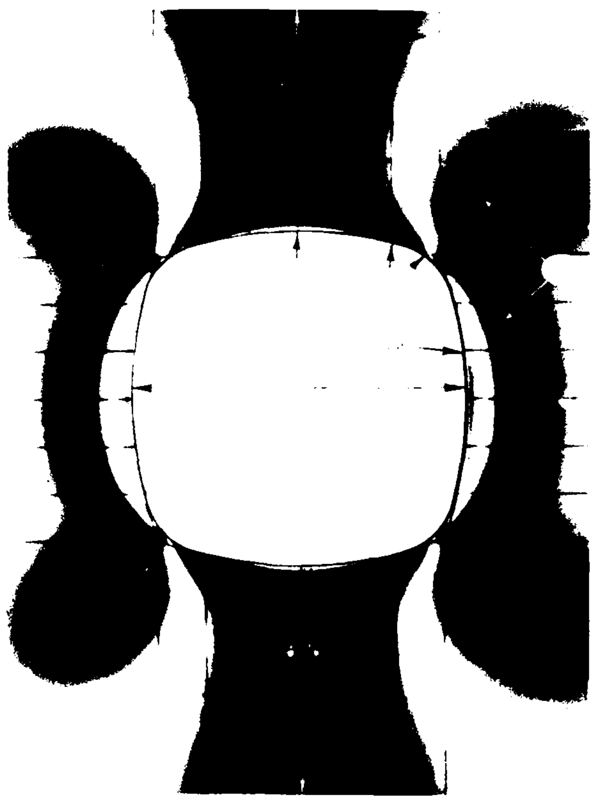

Fig. 7 Stresses at the boundary of a double barrel hole in a large plate (D/W = 0.163) subjected to uniaxial load

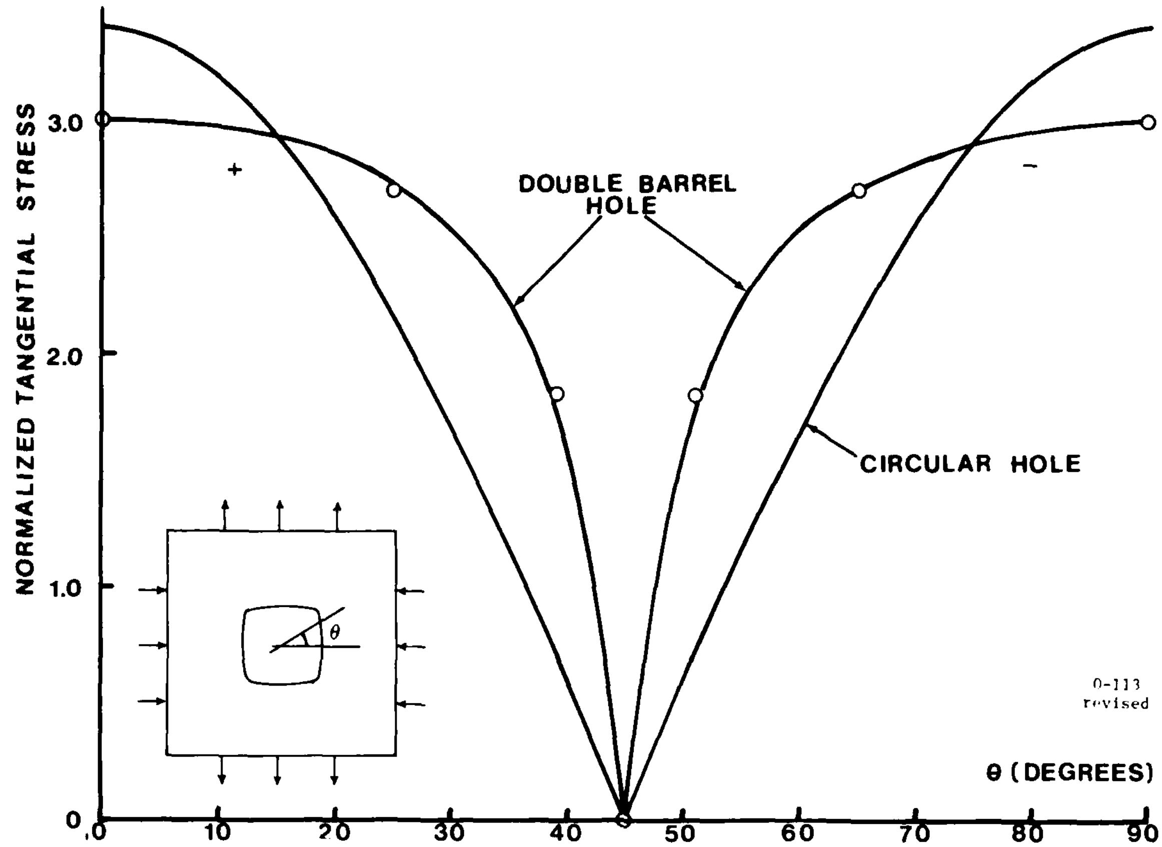

Fig. 8 Distribution of stress around a double barrel hole in a large plate (D/W = 0.163) subjected to biaxial loading of 1:-1 (pure shear)

minimum s.c.f., 0.67 for Richmond's minimum s.c.f. shape and 0.74 for Ross results obtained using Heywood's 'ideal shape.' The corresponding value for a circular hole is 0.61.

Richmond's values for s.c.f. are found to be significantly in error on the low side while Ross's results are significantly in error on the high side. It may be safely concluded that Ross's values for Heywood's "ideal shape" are also significantly overestimated; in fact, the s.c.f. for Heywood's shape may be expected to be less than 3. The isochromatic pattern given by Ross shows that Heywood's shape is quite close to an optimum with only a slight stress concentration on the horizontal axis of symmetry normal to the load. The radius of curvature for the longitudinal sides of the hole is given as D by Heywood while it is estimated to be 1.25D for the optimized shape proposed here.

For a plate subjected to pure shear, the double barrel shape yields a 10% reduction in s.c.f.

Acknowledgments

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This paper deals with the optimization of the shape of the corners and sides of a square hole, located in a large plate and subjected to in-plane loads, with the object of minimizing stress concentrations. Appreciable disagreement has been found between the results obtained previously by other investigators. In this paper new tests have been conducted and discrepancies have been corrected. Using an optimization technique, the authors have developed a quasi square shape which introduces a stress concentration of only 2.54 in a uniaxial field, the comparable

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) value for the circular hole being 3. The efficiency factor of the proposed optimum shape is 0.90 whereas the efficiency factor of the best shape developed previously was 0.71. The shape also is developed that minimizes the stress concentration in the case of biaxial loading when the ratio of biaxiality is 1:-1.

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TABLE OF CONTENTS

OPTIMIZATION OF INNER AND OUTER BOUNDARIES OF BEAMS AND PLATES WITH HOLES

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This paper presents optimized shapes of inner and outer boundaries for three specific problems: a long rectangular plate with a central hole subjected to uniaxial tension, a simply-supported slotted beam subjected to a load uniformly distributed over a small area at the centre, and a square plate with a central hole under uniaxial uniform pressure. The two-dimensional photoelastic method is used for optimization. The results indicate a significant reduction in stress concentration factor or in weight, or in both. The examples presented also include cases where the inner and outer boundary stresses are mutually dependent.

1 INTRODUCTION

Free boundaries of loaded components are usually subjected to stresses in the plane tangent to the boundary. In general these stresses vary from point to point. When a boundary has an optimum configuration all its points are subjected to the same value of stress. Optimization is the process by which the shape of a boundary is changed, within design constraints, to reach the situation in which all points, or most of the points, of the boundary have the same value of stress, which is a minimum for a particular situation.

Optimized shapes of holes located in long rectangular, square and circular plates under specified loading conditions have been presented recently in a series of papers (1)–(5).† The optimization process involves the use of two-dimensional photoelasticity in a systematic way. When permitted by the constraints of the problem, it is possible to optimize, in many instances, not only the inner boundary, but the outer boundary as well. Depending on the configuration and loading conditions, the inner and outer boundary stresses can be mutually dependent or independent. When they are independent of each other's influence, it is straightforward to follow the optimization process presented in references (1) (5). When they are mutually dependent, the optimization process becomes more involved, requiring operation on both of the boundaries alternately. There are also favourable situations in the second case when, as the optimization process is carried out on one of the boundaries, the other boundary is automatically optimized due to mutual dependency. It is the purpose of this paper to present examples of these cases with results obtained from the analysis of beams and plates with holes.

2 OPTIMIZATION PROCEDURE

The method consists of using photoelasticity in a systematic way to idealize a configuration so that no gradients of stress occur along the length of the boundaries. In other words, the structure will have stresses uniformly, or almost uniformly, distributed along the boundaries. The geometric constraints for the problem are stipulated initially. The procedure permits the direct design of the geometry of the structure rather than the conventional step-by-step design and analysis, satisfying the requirement that the maximum stress should be lower than an allowable stress, whilst at the same time the distribution should be as efficient as possible. A transparent model of the structure is placed in a diffused light circular polariscope. (The material of the model should exhibit birefringence when under load and should be sufficiently sensitive to produce several fringes of interference.) The operator should be able to work on the model with a hand file or portable router while studying the model through the analyzer. The method requires that the operator file away material from the boundaries starting at the points where the stress (and therefore the fringe order) is at a minimum. The constraints specified initially determine the amount of material the operator can remove. The removal of material redistributes the stresses and hence, the fringes. The operator continues filing away material from the low stressed zones of the boundary until as much as possible of the length of the boundary shows the same order of birefringence. This is easy to detect because at that moment, the same isochromatic fringe falls all along the length of the boundary. If the body has more than one boundary, it may be necessary to operate by steps back and forth from one boundary to the other. In some cases, white light may be more practical than monochromatic light, the objective being then to have the same colour along the boundary.

The degree of optimization can be evaluated quantitatively by a coefficient of efficiency

$$k_{eff} = \frac{1}{S_2 - S_0}\left(\frac{\int_{S_0}^{S_1} \sigma_t \cdot ds}{\sigma_{all^*}} + \frac{\int_{S_1}^{S_2} \sigma_t \, ds}{\sigma_{all^-}}\right)$$

where σ_t is the tangential stress, σ_{all} represents the

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† References are given in the Appendix.

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maximum allowable stress (the positive and negative superscripts referring to tensile and compressive stresses, respectively). S_0 and S_1 are the limiting points of the segment of boundary subjected to tensile stresses, and S_1 and S_2 are the limiting points of the segment of boundary with compressive stresses. The significance of the coefficient of efficiency is discussed in references (2) and (4). The above criterion will be used in the present work to evaluate the optimized boundaries.

The design procedure will be particularly useful for components made with brittle materials or components made with ductile materials subjected to fatigue.

In this paper, using the technique described above, optimized inner and outer boundary shapes are presented in: (i) a long rectangular plate with a central hole subjected to uniaxial tension, (ii) a simply supported slotted beam subjected to a load distributed over a small area, and (iii) a square plate with a central hole under uniaxial uniform pressure.

3 EXPERIMENTAL DETAILS

Experiments were conducted with 0.24 in (6 mm) thick Homalite-100 plates (fringe constant of 156 lb/in-fr (27.3 kN/m-fr)). Under loaded conditions, material was removed from the low stress regions by careful use of a hand held router, the final finishing being achieved with a hand file, respecting all the time the geometric constraints specified. To improve the precision, in particular at the corner zones, the operator used, during the filing process, a binocular magnifier with a set of polarizing and quarter wave plates attached to each of its lenses. This permitted the operator to see the fringes directly while working on the model. The constraints specified, the model used, and the results obtained for the three problems mentioned above are given below.

3.1 Long rectangular plate with a central hole subjected to uniaxial tension (eye bar problem)

It is specified that the stress should be constant all along the outer tensile boundary and also constant along the tensile and compressive segments of the inner boundary. The geometric constraints are: (i) the central hole boundary should lie in between a square of side D and a circle of diameter D, and (ii) the outer boundary should be optimized keeping same hole diameter to plate width ($= D/W$) ratio.

Optimization of inner and outer boundaries has been carried out on a 24 in (610 mm) long by 3.5 in (89 mm) wide plate for two typical D/W ratios, $D/W = 0.56$ and 0.87. The isochromatic patterns for these two cases are shown in Figs 1 and 2. The stress distributions around the inner hole boundary are given in Fig. 3. The experimentally determined optimum inner and outer geometries have been fitted with a combination of circles of different diameters and common tangents at the points of intersection. The geometries for the two D/W ratios are illustrated in Figs 4 and 5.

It is seen from the isochromatic patterns in Figs 1 and 2 that the hole boundary as well as the outer boundaries exhibit a high degree of optimization with the stresses

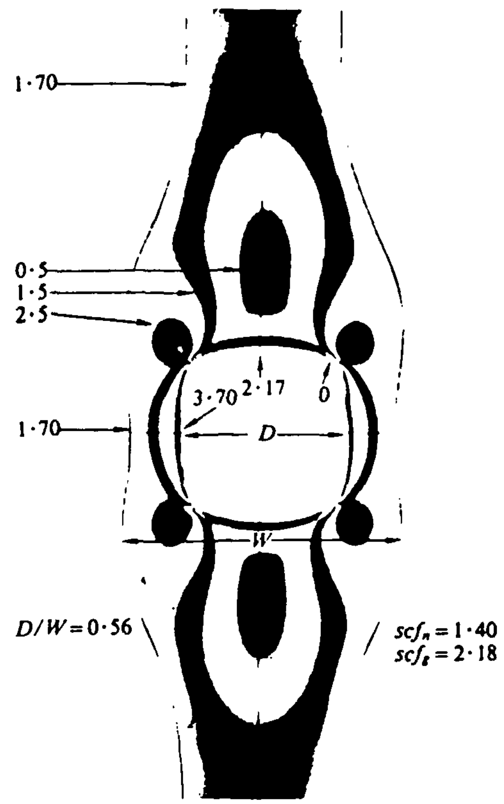

Fig. 1. Optimization of the inside and outside boundaries of an eye bar subjected to axial load ($D/W = 0.56$)

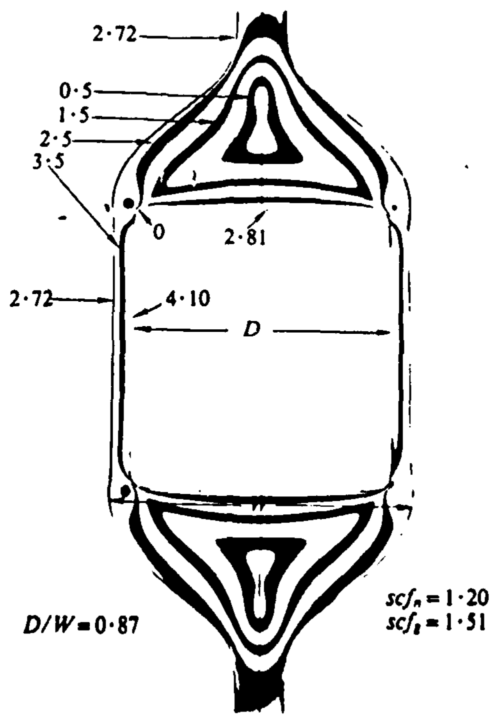

Fig. 2. Optimization of the inside and outside boundaries of an eye bar subjected to axial load ($D/W = 0.87$)

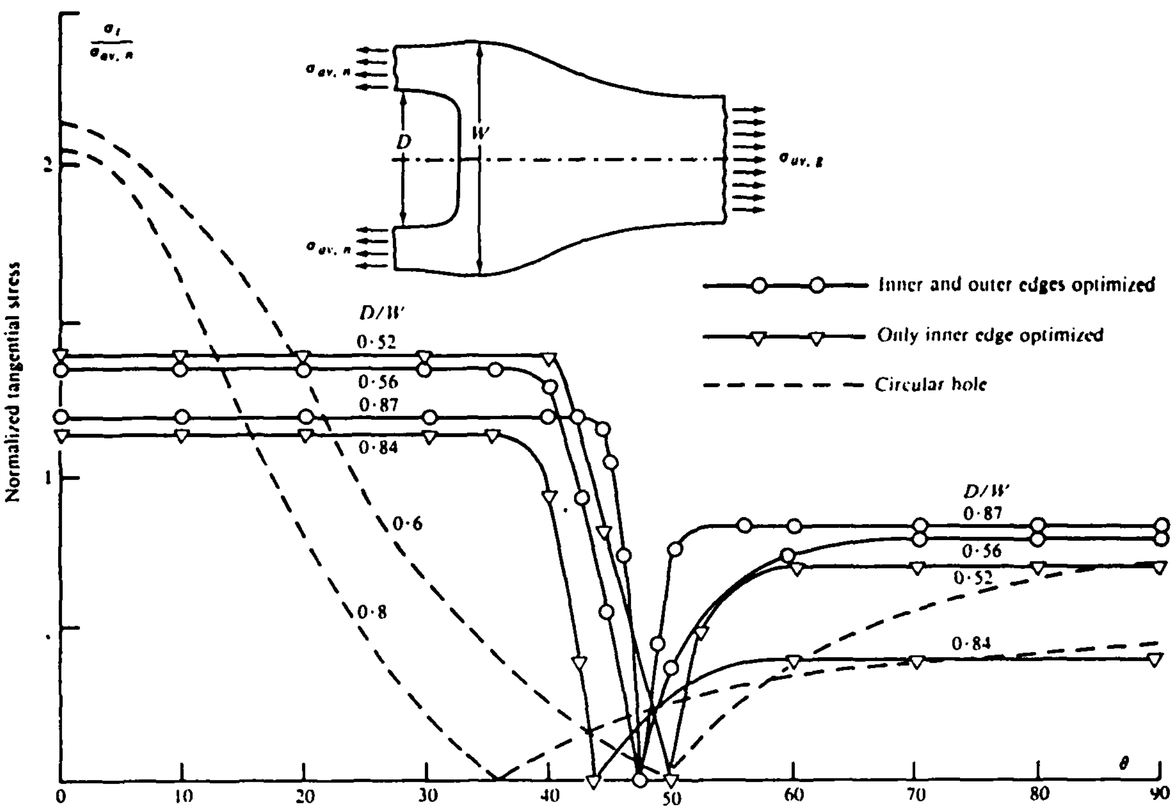

Fig. 3. Stress distribution at the boundary of an optimized hole in an eye bar

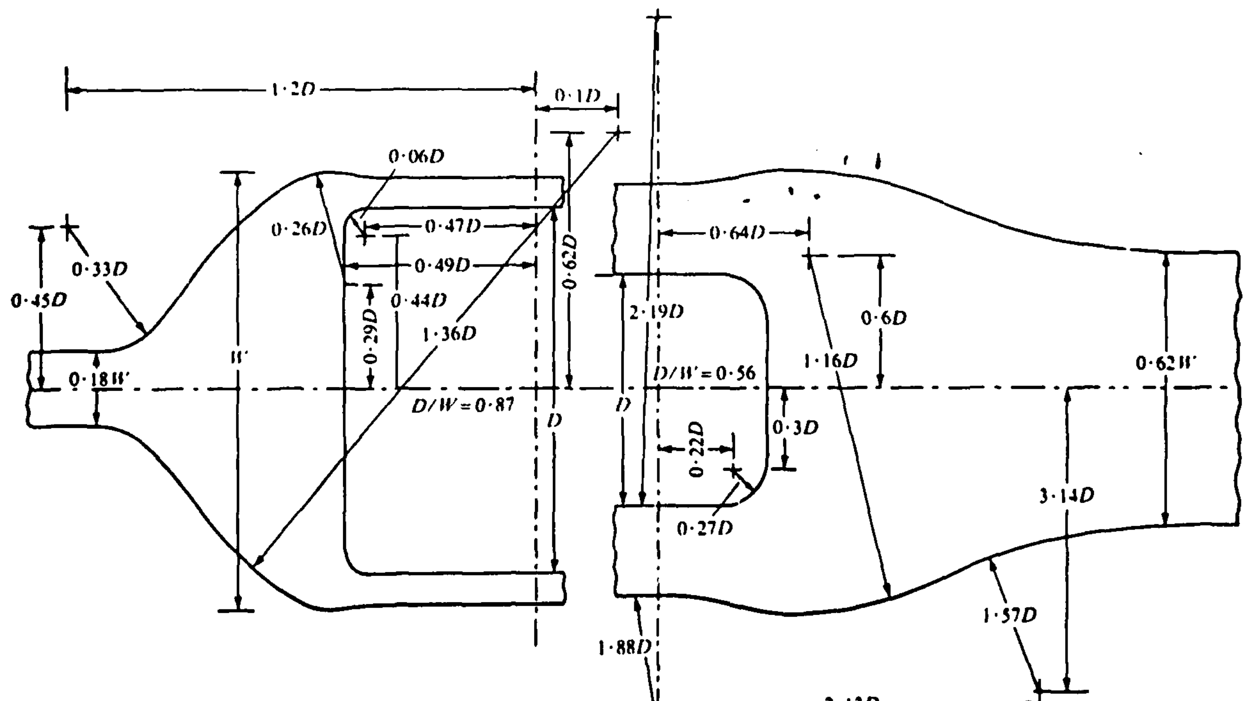

Fig. 4. Geometry of eye bars with optimized inside and outside boundaries (D/W = 0.56 and 0.87)

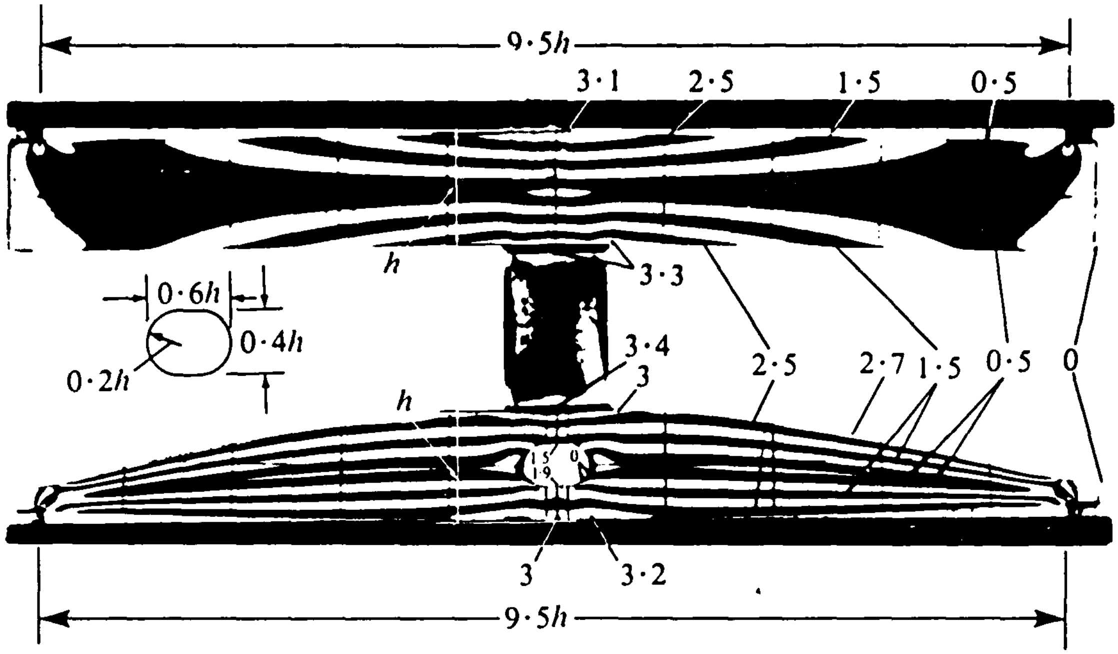

Fig. 5 Isochromatics in: (i) a uniform beam simply supported and loaded at the centre, and (ii) an equally loaded, optimized slotted beam of the same length, height, and maximum stress

remaining uniform along the outer boundaries as well as on the tensile and compressive segments of the inner boundary. The weight reduction for the specimens with $D/W = 0.56$ is about 13 per cent in comparison with a rectangular plate with a central circular hole having the same D/W ratio and a length of five times the hole width. For the case of $D/W = 0.87$, the weight reduction is about 33 per cent. In both the cases, a coefficient of efficiency of about 0.96 has been achieved for the inner and outer boundaries. The reduction in the stress concentration factor as compared to a rectangular plate with a circular hole having the same D/W, is about 34 per cent for $D/W = 0.56$ and 40 per cent for $D/W = 0.87$.

Comparison of the present results with those given earlier for $D/W = 0.52$ and 0.84 in reference (3) for the case of optimized inner boundaries with the outer boundaries kept straight brings out certain interesting features. For the case of D/W equal to about 0.56, the shape of, and the stress distribution around, the inner hole boundary remain virtually unaffected by the optimization of the outer boundary (Fig. 3). However, for the case of D/W equal to about 0.87, both the shape and the stress distribution around the hole edge get modified. Even here, while the tensile straight vertical edge with its stress concentration factor equal to 1.2 remains unaffected, there are drastic changes occurring on the compressive segment of the hole boundary. The shape of this segment has changed from that of a hat to almost a straight line, while the value of compressive stress concentration factor has gone up from 0.4 to 0.84.

One of the constraints of the problems, as solved, is that the stress at the outside boundary should remain constant. In some design applications it may be more useful to specify that the average stress at the top and bottom portions of the bar be the same as the maximum stress at the inside of the longitudinal boundary. This could easily be obtained by reducing further the width of the bar at that point.

The configuration shown in Fig. 1 can also be seen as the cross-section of a long wall subjected to longitudinal compression. In this case, besides the fact that the boundaries subjected to compression have all the same value of stress, the transverse boundary under tension exhibits a stress about 40 per cent smaller than the compressive one, a requirement which corresponds to the ratio of strengths in many brittle materials

3.2 Simply-supported slotted beam subjected to load uniformly distributed over a small area at the centre

In this example, specified constraints are: (i) the lower boundary of the beam should remain straight, (ii) the length of the slot should not be longer than 0.15 times the length of the beam, and (iii) the maximum stress on the inner boundary should not be higher than the maximum stress on the outer boundary.

A beam of dimensions 10 in. × 1 in. (254 mm × 25.4 mm) was chosen and subjected to a central load distributed over a small area. A single central slot in the shape of an ovaloid of 0.4 in × 0.6 in (10 mm × 15 mm) size was introduced into the beam initially, and the outer and inner boundaries were optimized within the constraints specified. The isochromatic pattern presented in Fig. 5 shows the results obtained for this case along with the case of a solid beam of uniform cross-section under identical loading conditions. The hole height shown is about the maximum attainable before the stresses at the outer boundaries exceed the level of the maximum stresses of the solid beam. It is seen from Fig. 5 that as well as the outer boundaries, the major portions of the inner hole boundary have constant uniform stresses. It is also found that the outer boundary shape is virtually the same as the optimized shape given by the strength of materials theory for a solid beam for the same loading.

Comparing the present results with those for a simply-supported solid beam of uniform depth under identical loading conditions, the maximum stresses are virtually the same in both cases. However, the weight of the beam has come down by as much as 31 per cent. Further decreases in weight could be obtained if the constraints permit extension of the length of the hole toward the support of the beam. A coefficient of efficiency of about 0.93 has been maintained for the outer boundary while it is about 0.81 for the inner boundary. The maximum stress on the inner boundary is only about 60 per cent of maximum stress on the outer boundary. The stress on the inner boundary could be further optimized if desired. If the requirement that the bottom of the beam should remain straight is relaxed and allowed a slight curvature, the stress can be made constant all along the boundary. The maximum deflection in the optimized beam is about 27 per cent larger than the deflection in the solid beam.

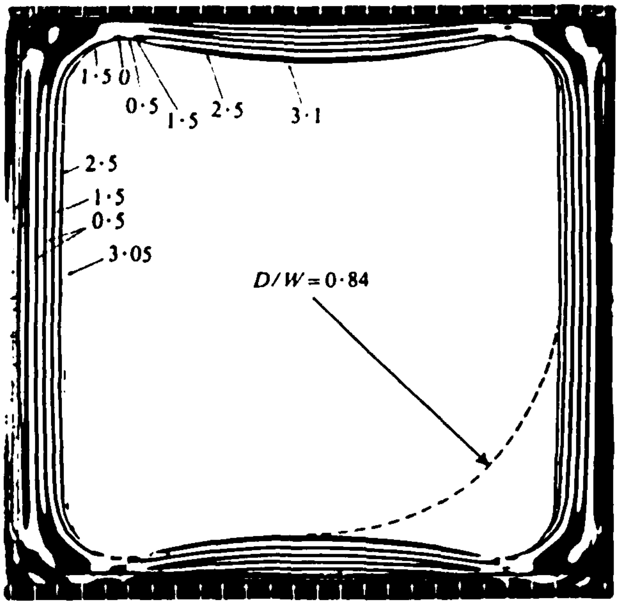

Fig. 6. Typical isochromatic pattern around an optimized hole in a square plate subjected to uniform pressure on two opposite sides

3.3 Square plates with central optimized hole under uniaxial uniform pressure

In the first example, the case of mutual dependency as well as the case of independency of stresses at the two boundaries was presented. When these stresses are mutually dependent, the change in shape of one boundary is associated with a change in stress in the other boundary, and the optimization may require a change in shape of that boundary also. In the first example given above, the stress on the other boundary could not be brought down. In the example that follows, a case will be shown wherein the mutual dependency works favourably.

The constraints specified for this case: (i) the outer boundary should remain square, and (ii) the inner boundary should clear a circle of diameter D. With these constraints, a plate 3 in. × 3 in. (76.2 mm × 76.2 mm) was chosen and subjected to a uniform uniaxial compressive load. The complete details of the set up and results can be found in reference (5). Here the results for the case of $D/W = 0.84$ will be considered for discussion.

The isochromatics for the optimized hole are shown in Fig. 6. The stress distribution on the outer boundaries for the optimized as well as the corresponding circular holes are presented in Fig. 7. It is found that the optimization of the inner boundary has lead to the optimization of the unloaded outer boundary as well, with significant reduction in the peak stress. The resulting coefficient of efficiency for that boundary is about 0.88, which can be further improved if one relaxes the first constraint. Even on the loaded boundary, the tangential stress distribution is found to be more uniform. It may be mentioned that a similar effect was observed for the case of thin circular rings during the optimization process (4).

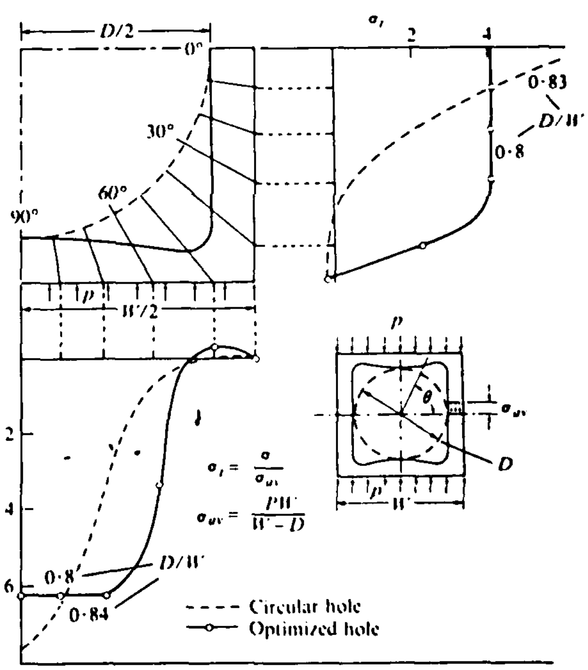

Fig. 7. Stress distribution along the outer boundaries of a square plate with an optimized hole subjected to uniform pressure on two opposite sides

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APPENDIX

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This paper presents optimized shapes of inner and outer boundaries for three specific problems: a long rectangular plate with a central hole subjected to uniaxial tension, a simply-supported slotted beam subjected to a load uniformly distributed over a small area at the centre, and a square plate with a central hole under uniaxial uniform pressure. The two-dimensional photoelastic method is used for optimization. The results indicate a significant reduction in stress concentration factor or in weight, or in both. The examples presented also include cases where the inner and outer boundary stresses are mutually dependent.

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